CHAPTER 3

GLOBAL ILLUMINATION PROBLEM FORMULATION

One of the consequences of using physical laws for modeling light transport is that we must be at least a little careful about what quantities are used. In Section 3.1, some useful physical quantities are introduced, including *power* and *radiance* which are perhaps the most useful quantities for computer graphics simulations. Section 3.2 considers the radiance at all points and directions in a room to be a function, and frames the computer graphics problem in terms of evaluating this function. Some care is taken to make sure the function is defined at all points including points exactly on surfaces. Some properties of the function are also discussed. Section 3.3 develops the *rendering equation* which describes light interaction at a surface. This development of the rendering equation differs from that of previous authors by including a precise treatment of transparent surfaces[56, 61]. Finally, Section 3.4 summarizes the main points of this chapter.

In the interest of simplicity, this chapter will ignore the possibility of a participating medium such as smoke. This omission will be corrected in Chapter 7.

3.1 Terms and Units

In this section some important terms are defined and described. In particular, the most important physical quantities used in realistic graphics, *power* and *radiance*, are informally introduced. The vocabulary and notation introduced in the section are based closely on the standard terms and notation used in the field of Illumination Engineering. The reason this terminology was chosen over that of the field of Heat Transfer is that the Illumination Engineering community has adopted a standard terminology[57]. This terminology is also commonly used in Physics[31]. This may cause some confusion because much of the Computer Graphics literature uses Heat Transfer notation, notably the groundbreaking papers from Cornell University[40, 23, 24, 56, 93, 117]. Unfortunately, the field of Heat Transfer has no universal standard notation, so it has been avoided in this work. Readers familiar with Heat Transfer notation should note that Heat Transfer's *intensity* is Illumination Engineering's *radiance*.

As discussed in the Chapter 2, the geometrical optics approximation allows us to think of optical transport as noninteracting rays bouncing around a scene that is to be rendered. The fundamental unit that measures the energy of a packet of rays is *radiant energy*, denoted by Q.

The radiant energy is simply a measure of light energy. Since we are interested in the amount of light hitting a surface or film plane during a set time period, *radiant power* (also called *radiant flux*), the radiant energy per unit time, Φ, is often used. Henceforth I will refer to radiant power simply as *power*. Power is often convenient to work with because it allows energy balance constraints to be easily applied[1].

[1] Energy balance constraints are only easy to apply if the solution is steady state or we assume that the speed of light is infinite. Otherwise we have to allow for the lengths of light ray paths. Since the time it takes light to travel across a typical scene (such as a living room) is very small compared to a camera shutter speed or the human temporal visual threshold, assuming infinite light speed is usually appropriate.

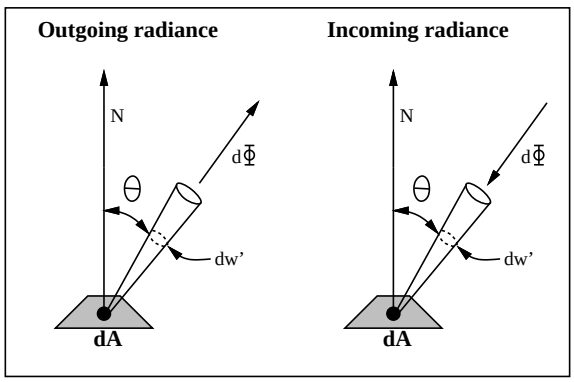

Figure 3.1: Geometry for Definition of Radiance.

Power is sometimes not a natural quantity to use for graphics. For example, we characterize the appearance of an object by its 'color'. Unlike power, this notion of object color can depend on viewing angle, as is true for a mirror. The amount of power traveling from a source in a certain direction can be measured as *radiant intensity*, $I = d\Phi/d\omega$, where ω is the solid angle originating at the source.

One characteristic of radiant intensity is that it depends on the area of the light source. This is not true of object color, which is independent of surface area. The quantity that more closely approximates color is the *radiance*, L (see Figure 3.1):

$$L = \frac{d^2\Phi}{d\omega dA cos\theta}$$

Or in terms of radiant intensity, radiance is:

$$L = \frac{dI}{dA cos\theta}$$

The radiance gives an indication of surface brightness, dependent upon neither the size of the object being viewed, nor the distance to the viewer. It usually suffices to think of radiance as the power passing through a point from a unit solid angle (fraction of the visual field). The radiance at a certain point $\mathbf{x}$ in a direction ψ is denoted $L(\mathbf{x}, \psi)$. A *direction* is just a vector, or can be thought of as a (θ, ϕ) pair in spherical coordinates.

These quantities (energy, power, radiant intensity, and radiance) as described above are taken across many optical wavelengths. Since we eventually want to find visual colors, the wavelength breakdown of these quantities is needed. Instead we can think of them as functions of wavelength by measuring them *per unit wavelength*. These new wavelength dependent quantities are called *spectral energy*, *spectral power*, *spectral radiant intensity*, and *spectral radiance*. Since wavelength dependent information is always needed for graphics, the word *spectral* should be assumed to be implicit for the remainder of the text.

3.1.1 Reflectance Terms and Units

A simple way to describe the reflectance of a surface is by the absolute reflectance $R(\psi_{in}, \lambda)$, the fraction of light at wavelength λ incident from direction ψ_{in} that is not absorbed. The reflectance is often too simple a measurement because the distribution of the reflected light is not described. To overcome this shortcoming, the bidirectional reflectance-distribution function (**BRDF**), ρ, can be used. Written in terms of radiance the expression for the bidirectional reflectance is:

$$\rho(\mathbf{x}, \psi_{in}, \psi_{out}, \lambda) = \frac{dL(\mathbf{x}, \psi_{out}, \lambda)}{L(\mathbf{x}, \psi_{in}, \lambda) d\omega_{in} \cos\theta_{in}} \tag{3.1}$$

Here, $\mathbf{x}$ is the point of reflection, ψ_{in} is the incident direction, ψ_{out} is the direction of reflectance, $d\omega_{in}$ is the differential solid angle the incoming light arrives through, and θ_{in} is the angle between ψ_{in} and the surface normal at $\mathbf{x}$.

The definition of the **BRDF** is not as mysterious as might first appear. The numerator $dL(\mathbf{x}, \psi_{out}, \lambda)$ is the radiance of the surface seen from a point in direction ψ_{out}. The denominator $L(\mathbf{x}, \psi_{in}, \lambda) d\omega_{in} \cos\theta_{in}$ is simply the incident power per unit area. This makes the measurement of the **BRDF** easy in theory. But in practice data for the **BRDF** of real surfaces is hard to

come by. Methods of measuring **BRDF** can be rather tedious and expensive, so this lack of data is not surprising[76]. There is some hope that this data shortage is not permanent; an apparatus that will allow cheaper measurement of the **BRDF** of real surfaces has been designed at Lawrence Berkeley Lab, and initial results are promising[91].

An important property of **BRDF**s is called the *Helmhotz Reciprocity Rule*, which states that the **BRDF** is symmetric relative to ψ_{in} and ψ_{out} (REF). Quantitatively, the rule can be stated:

$$\rho(\mathbf{x}, \psi_{in}, \psi_{out}, \lambda) = \rho(\mathbf{x}, \psi_{out}, \psi_{in}, \lambda) \tag{3.2}$$

The reciprocity rule is examined theoretically and verified experimentally in [21].

Sometimes it is more convenient to work with the radiant power Φ than with the radiance L. On these occasions the **BRDF** is cumbersome. It is more natural to view the surface reflection properties in terms of the probability distribution of the reflected light. This can be called the *scattering probability function* (**SPF**), s:

$$s(\mathbf{x}, \psi_{in}, \psi_{out}, \lambda) = \frac{dI(\mathbf{x}, \psi_{out}, \lambda)}{R(\mathbf{x}, \psi_{in}, \lambda) d\Phi(\mathbf{x}, \psi_{in}, \lambda)}$$

The **SPF** directly describes the amount of energy scattered in each direction ψ_{out}. The term $R(\mathbf{x}, \psi_{in}, \lambda)$ appears in the denominator to scale the function to a valid probability density function over the solid angles ω. Thus the probability of an energy packet of wavelength λ incident on point $\mathbf{x}$ from direction ψ_{in} being scattered in direction ψ_{out} is $R(\mathbf{x}, \psi_{in}, \lambda) s(\mathbf{x}, \psi_{in}, \psi_{out}, \lambda) d\omega(\psi_{out})$, and the probability of it being absorbed is $(1 - R(\mathbf{x}, \psi_{in}, \lambda))$. The **BRDF** and the **SPF** have the simple relationship $\rho = C s \cos(\theta(\psi_{out}))$, where C is the constant that enforces the unit area constraint for a probability density.

3.2 Image and Global Radiance Functions

At a point $\mathbf{x}$ in an environment, such as a room, there is a well defined radiance value for every direction ψ and wavelength λ. This radiance can be denoted $L(\mathbf{x}, \psi, \lambda)$, which for our purposes can be thought of as a function varying over points $\mathbf{x}$ in the scene to be rendered, directions ψ, and visible wavelengths λ. This function is called the *Global Radiance Function*, and it incorporates the movement of all light through an environment. Because light bends at the surfaces of objects (Snell's Law, Section 2.2), L is *not* defined at a point exactly on a surface. This is an easy problem to get around for opaque surfaces, but is more troublesome for transparent surfaces. For this reason, some care will be taken in treating the function on surfaces.

We can look at calculating the image function, I, in terms of evaluating the global radiance function at the viewer position. This is because all of the considerations about the image are present in $L(\mathbf{x}_{eye}, \psi_{view}, \lambda_{vis})$, where $\mathbf{x}_{eye}$ is the eye position, ψ_{view} varies over all directions within the view frustum, and λ_{vis} varies over the visible wavelengths.

The global radiance function has a basic property that is used explicitly or implicitly in most graphics algorithms: L is constant along a line uninterrupted by objects. This property implies that as we change our distance to an object, but not our viewing direction, the radiance we measure does not change. Intuitively, this just means that color does not change with distance, which accords with everyday experience. This property can be formulated as the *Ray Law* (Figure 3.2), which states that L is constant along a ray from $\mathbf{x}$ to the first surface hit by the ray:

$$L(\mathbf{x}, \psi, \lambda) = L(\mathbf{x} - t\psi, \psi, \lambda) \quad \text{if } 0 < t < t_0$$

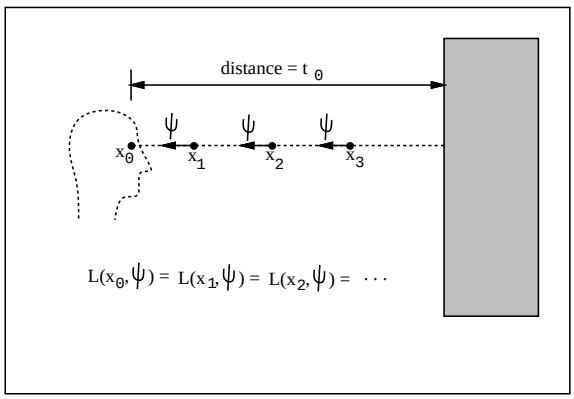

Figure 3.2: The radiance L stays constant along a line of sight.

where t_0 is the distance to the first surface seen from $\mathbf{x}$ in direction ψ (this assumes that ψ is a unit vector). This point is usually determined by sending (tracing or casting) a *ray* from $\mathbf{x}_0$, in direction ψ, until some surface is hit.

Though the idea of the Global Radiance Function sounds elegant, as defined above it is not complete enough to be useful. The Ray Law implies that once we know the radiance at all surfaces, we can 'fill in' the values between surfaces. Unfortunately, the Global Radiance Function is only defined between the surfaces! The most straightforward way to handle this problem is to push the definition of the GRF to surfaces by taking a limit. For opaque surfaces taking this limit is easy because we do not need a radiance to be defined inside the surface. In Figure 3.3 this process is shown, where the radiance at the surface is defined to be the limit as point x is pushed along some path until the distance to the surface is zero. This will insure continuity of L *as measured from above the surface.* This limit is often implicitly assumed when taking about the radiance coming out from a point, or into a point[56, 25].

At a transparent surface, this simple limit idea breaks down. Figure 3.4 illustrates that the limit can be taken from either side of the surface. The limit value will be different on each

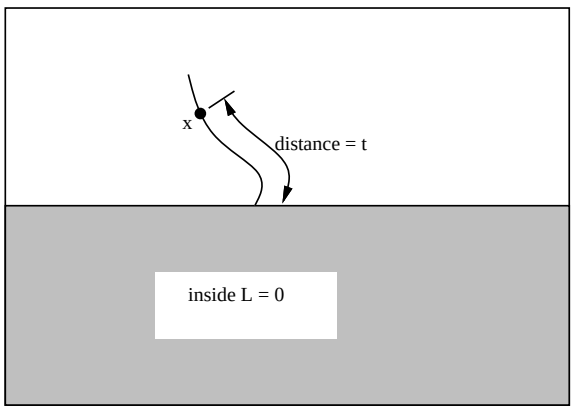

Figure 3.3: The radiance at x can be defined at the surface by taking the limit as t goes to zero.

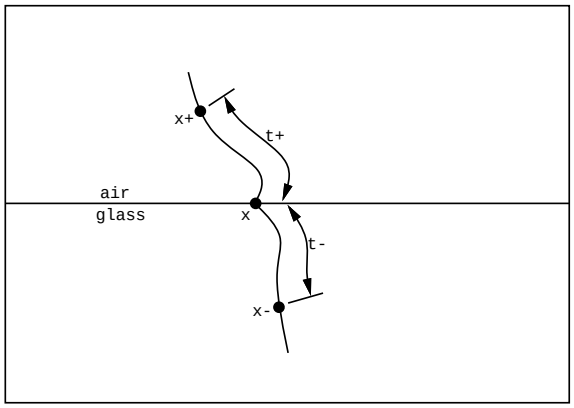

Figure 3.4: The radiance at the surface will be different if the limit is taken from x- instead of x+.

side[2], so L cannot be uniquely defined at the surface without prioritizing 'inside' and 'outside', a concept that does not make sense for the interface between glass and water in a fishtank, for example. One solution would be to always have a point between surfaces when evaluating L at a surface. This point would determine a 'inside' and 'outside' so that the limit could be taken from the side of that point.

[2]The electromagnetic field *is* constant at the surface boundary, which is the basic assumption of the Fresnel Equations. Snell's Law implies that light bends instantaneously at a surface, so its direction of propagation is discontinuous at the surface, though its electromagnetic field maintains continuity.

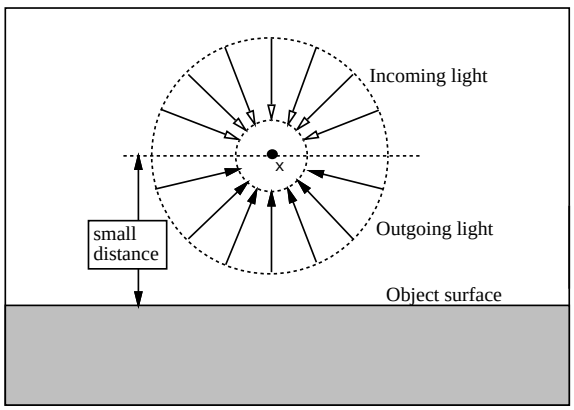

Figure 3.5: The sphere of directions just above a surface can be divided into incoming and outgoing directions.

Another way to take the limit of L at a surface is to divide the set of directions into 'incoming' and 'outgoing' directions as shown in Figure 3.5. In this picture, the black arrows indicate directions that light travels *away* from the surface. The white arrows show incoming directions. The radiance of the surface as measured by a viewer will always be $L(\mathbf{x}, \psi_{out})$, where ψ_{out} is one of the outgoing directions.

On one side of a transparent surface, a viewer will see $L(\mathbf{x}^+, \psi_{out}^+)$ and on the other side the viewer will see $L(\mathbf{x}^-, \psi_{out}^-)$, where ψ_{out}^+ and ψ_{out}^- make up two nonoverlapping hemispheres of directions. We can define the union of these two functions to be the unique limit of L at a surface. This will ignore the radiance values at a surface for *incoming* directions. To describe these values, a separate function L_{in} can be used. L_{in} is defined only at surfaces, and is made up of the incoming directional radiance values not used for L_{out}. This idea is shown visually in Figure 3.6. We could just as easily have used L_{in} for surface values of L, and considered L_{out} as a separate function. L_{out} is a more natural choice because the viewer 'sees' L_{out}, while L_{in} is needed only for reflection and transmission calculations.

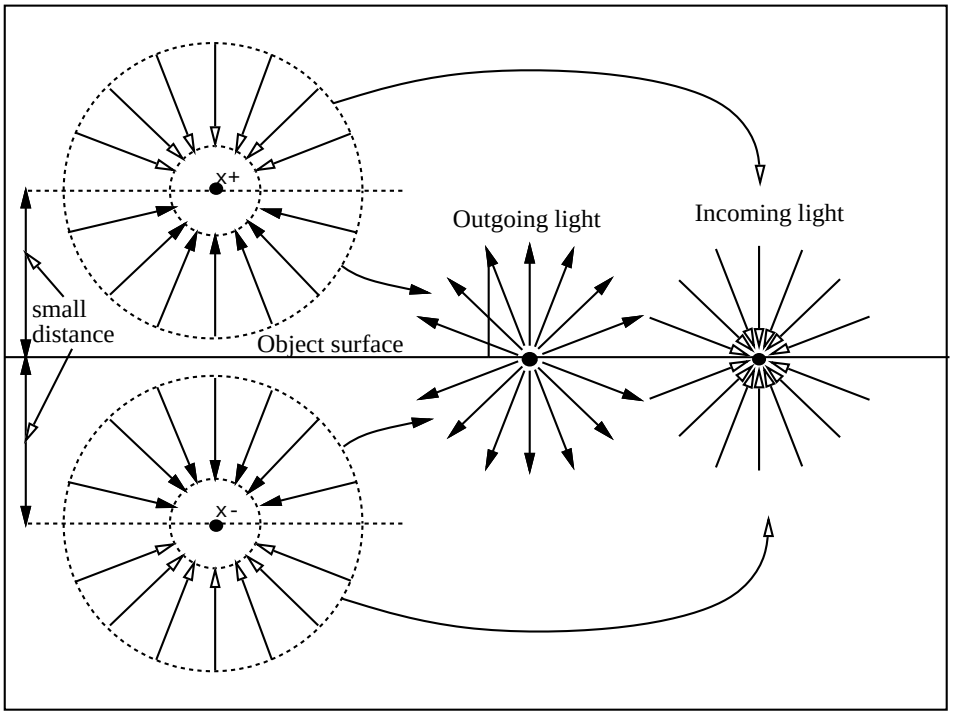

Figure 3.6: Incoming and outgoing radiance distributions at the surface are constructed from piecing together functions on either side of the surface.

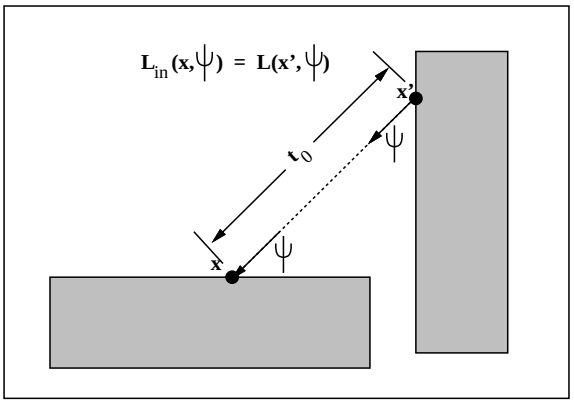

Figure 3.7: The incoming radiance at x is the radiance coming from another point.

Given the concept of L_{out}, we can extend the the Ray Law to include values $(0 < t \leq t_0)$. If $\mathbf{x}$ is not on a surface, then the Ray Law is correct for $(0 \leq t \leq t_0)$. If $\mathbf{x}$ *is* on a surface, then we can write down an expression for L_{in}:

$$L_{in}(\mathbf{x}, \psi, \lambda) = L_{out}(\mathbf{x} - t_0\psi, \psi, \lambda)$$

Here t_0 is again the distance to the first point seen from $\mathbf{x}$ in direction $-\psi$. The geometry of this idea is shown in Figure 3.7. Because L at surfaces is defined to be L_{out}, the equation could be also be written:

$$L_{in}(\mathbf{x}, \psi, \lambda) = L(\mathbf{x} - t_0\psi, \psi, \lambda)$$

This equation is most of the basis for most computer graphics lighting models. In the next section, adding some reflection properties to the surface will produce the rendering equation that forms a model for almost all computer graphics illumination methods.

The continuity of L depends on the continuity of the surfaces and on the continuity of the light emitted from surfaces. Assuming that the light emittance distribution functions and light reflectance distribution functions are piecewise continuous, and that the surfaces themselves are piecewise continuous, then L will also be piecewise continuous. This can be seen by examing the continuity of $L(\mathbf{x}, \psi)$ holding each variable constant in turn. Holding the point,$\mathbf{x}$, constant, L is the radiance seen in all directions from that point. This will be the radiance of surfaces seen from $\mathbf{x}$. Except at surface boundaries, these radiances will be piecewise continuous because the surface (and its reflectance and emittance) is piecewise continuous. Holding the direction, ψ, constant and varying the point, $\mathbf{x}$, also 'pans' across surfaces, so the same reasoning applies. Varying both variables in a continuous manner also 'pans', so the Global Radiance Function L is piecewise continuous. This continuity condition will not hold for fractal surfaces, but will apply to discretizations of such surfaces.

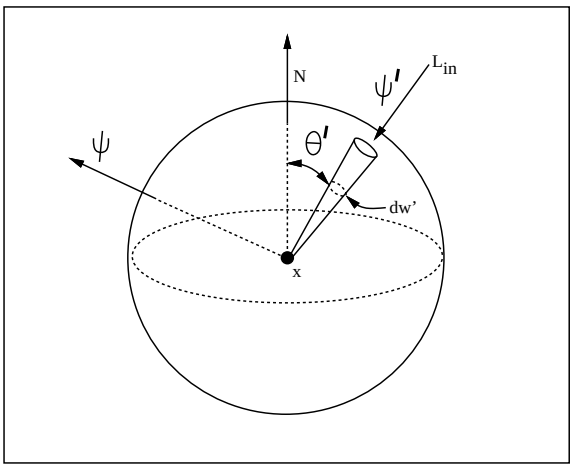

Figure 3.8: Geometry for Directional Rendering Equation.

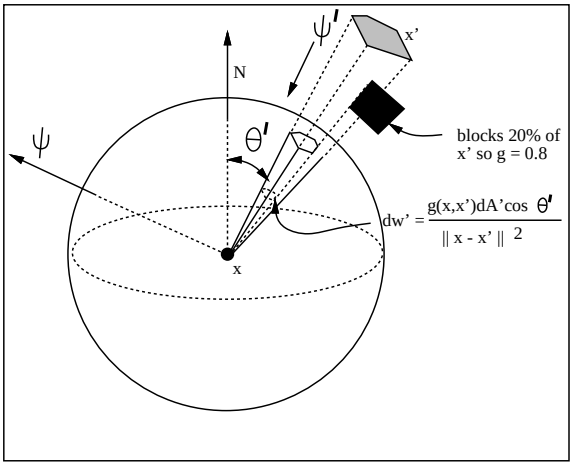

Figure 3.9: Geometry for Pointwise Rendering Equation. The black patch occupies 20% of the solid angle subtended by x'.

3.3 Rendering Equation

In the last section we saw that finding the value of $L(\mathbf{x}, \psi, \lambda)$ at a point $\mathbf{x}$ *not* on a surface is accomplished by finding the identically valued $L_{out}(\mathbf{x}_0, \psi, \lambda)$, where $\mathbf{x}_0$ is the point on the surface seen from $\mathbf{x}$ in direction $-\psi$. This can be further specified by dividing the surface radiance into reflected and emitted components:

$$L_{out}(\mathbf{x}_0, \psi, \lambda) = L_e(\mathbf{x}_0, \psi, \lambda) + L_r(\mathbf{x}_0, \psi, \lambda)$$

Here L_e is the emitted component of spectral radiance, and L_r is the reflected component. L_e is presumably known from input data, so the main computational problem is finding L_r. Since all the reflected light must come from the set of all incoming directions $\psi \in \bigcirc$, the expression for L_r comes straight from the definition of ρ (Equation 3.1). If direction ψ' is the only source of non-zero radiance, then,

$$L_r(\mathbf{x}, \psi, \lambda) = \rho(\mathbf{x}, \psi, \psi', \lambda) L_{in}(\mathbf{x}, \psi', \lambda) \cos\theta' d\omega'$$

If there are many incoming directions with non-zero radiance, then we must integrate over all incoming directions[3] that influence L_r. This yields:

$$L_{out}(\mathbf{x}, \psi, \lambda) = L_e(\mathbf{x}, \psi, \lambda) + \int_{\psi' \in \bigcirc} \rho(\mathbf{x}, \psi, \psi', \lambda) L_{in}(\mathbf{x}, \psi', \lambda) \cos\theta' d\omega' \tag{3.3}$$

The geometry for Equation 3.3 is shown in Figure 3.8.

The rendering equation is often called the *transport equation* in the heat transfer literature[54]. The recursive nature of this equation makes it non-trivial to solve. The form of Equation 3.3 is most similar to Immel et al.'s formulation of the rendering equation[56]. Kajiya's form differs in that it integrates over all surfaces rather than angles, and it is not written in terms of radiance[61]. Modifying Kajiya's representation to use radiance yields:

$$L_{out}(\mathbf{x}, \psi, \lambda) = L_e(\mathbf{x}, \psi, \lambda) + \int_{\text{all } \mathbf{x}'} g(\mathbf{x}, \mathbf{x}') \rho(\mathbf{x}, \psi, \psi', \lambda) L_{in}(\mathbf{x}, \psi', \lambda) \cos\theta' \frac{dA' cos\theta''}{\|x' - x\|^2} \tag{3.4}$$

Here x', with differential area dA', varies over all points of all surfaces, $g(\mathbf{x}, \mathbf{x}')$ is a geometry term that is one if x' can 'see' x and zero otherwise, and ψ' is the direction from x' to x. The geometry for this equation is shown in Figure 3.9.

[3] This means the definition of ρ must be extended to include all directions, including those below the plane of scattering.

Because the radiance arriving at $\mathbf{x}$ from $\mathbf{x}'$ is *outgoing* from $\mathbf{x}'$, Equation 3.4 can also be written:

$$L_{out}(\mathbf{x}, \psi, \lambda) = L_e(\mathbf{x}, \psi, \lambda) + \int_{\text{all } \mathbf{x}'} g(\mathbf{x}, \mathbf{x}')\rho(\mathbf{x}, \psi, \psi', \lambda)L_{out}(\mathbf{x}', \psi', \lambda)\cos\theta' \frac{dA' cos\theta''}{\|x' - x\|^2} \quad (3.5)$$

This form of the equation is convenient because it is expressed only in terms of the L_{out} of the surfaces.

Both of the equations above are of the form:

$$a(x) = b(x) + \int_{x' \in \Omega} k(x, x')a(x')d\mu(x')$$

Equations of this form are *Fredholm Equations of the Second Kind.*

3.4 Summary

The physical quantities power and radiance are useful for graphics. Radiance is used when the lighting at a point is considered because it is an intensive quantity. Power is used when energy transport or energy balance is of concern.

The global illumination problem can be thought of as a function evaluation problem, where the function is the Global Radiance Function, L, which is defined at all points and in all directions. The value of L at surfaces can be defined to be the outgoing radiance, L_{out}. The incoming radiance at a surface can be defined as a separate function L_{in}. This distinction is needed only because of transparent surfaces.

The Global Radiance Function obeys the Ray Law; the radiance is constant along a line of sight between objects. The Global Radiance Function is piecewise continuous if the surfaces in scene are themselves piecewise continuous.

On the basis of the Ray Law and the definition of **BRDF** , we can write an expression for the radiance of a point in terms of the radiances of other points. This *rendering equation* is a Fredholm Equation of the Second Kind, and can be written down as an integral over all directions seen from a point, or as an integral over all points that lie on surfaces.

CHAPTER 4

SURFACE REFLECTION MODELS

Although all surfaces obey Snell's Law at a *microscopic* level, complex small-scale structure can give rise to *macroscopic* reflection properties that are quite complex, as illustrated in Chapter 2. This macroscopic behavior can be described by the **BRDF** for that surface.

The two crucial parameters in the rendering equation derived in the last chapter are the distribution of surfaces, and the characteristics of the **BRDF** for the surfaces. In this chapter, several types of idealized **BRDF**s are discussed in the context of traditional computer graphics reflection models. The relationship of these models to the surface geometry classes presented in Chapter 2 is also discussed.

Section 4.1 describes the behavior of perfectly diffuse *Lambertian* surfaces, which are an idealized form of matte surfaces. That section also discusses diffuse transmitters, an idealized type of translucent surface. A new modification to the diffuse transmission model is also made in that section. In Section 4.2, perfectly smooth reflecting surfaces are examined, based on the discussion of Section 2.2. Surfaces that are part diffuse and part mirror are discussed in Section 4.3. This section presents an improved model of how such surfaces reflect light by including Fresnel Equations effects for the specular *and* diffuse terms. Section 4.4 describes reflection behavior that is more general than diffuse and specular behavior. In Section 4.5, the

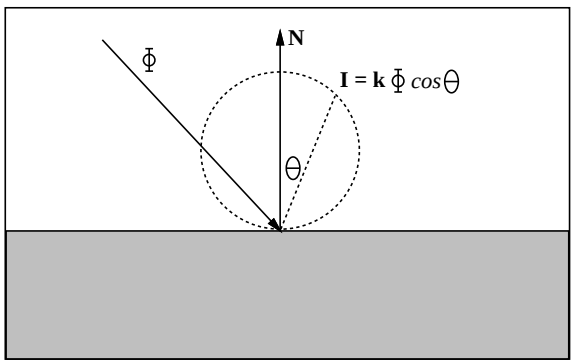

Figure 4.1: Ideal diffuse reflection.

principles described in the earlier parts of this Chapter are used to form guidelines for good parameter selection in traditional lighting models. Finally, Section 4.6 summarizes the content of this chapter.

4.1 Diffuse Reflection and Transmission

Although the reflectance characteristics of real surfaces are quite varied and complex, there are a few simple surface types that usually are used to approximate real reflection distributions. The most common surface type used in graphics is an idealized matte surface, the diffuse reflector. The diffuse reflector is sometimes called a *Lambertian* surface because it obeys Lambert's Law, which states that the **BRDF** is constant. The **BRDF** can only be constant if the numerator in Equation 3.1 is constant. This implies that a diffuse reflector has a constant spectral radiance at all viewing angles under steady lighting conditions. We can see that this is a decent approximation for many materials such as matte paint, which do not noticeably change *color* (the perceptual approximation to spectral radiance) as we change our viewpoint. This means the diffuse **BRDF** , ρ_d is given by the constant:

$$\rho_d(\psi_{out}, \psi_{in}, \lambda) = \frac{R(\lambda)}{\pi}$$

And the corresponding scattering distribution function, as shown visually in Figure 4.1, is:

$$s_d(\psi_{out}, \psi_{in}, \lambda) = \frac{\cos\theta}{\pi}$$

Of course, these equations for ρ_d and s_d are valid only for light incident and reflecting *above* the plane. Otherwise both ρ_d and s_d are zero.

The radiance for a diffuse reflector can be related to the power hitting it at point $\mathbf{x}$:

$$L(\mathbf{x}, \lambda) = \frac{R(\mathbf{x}, \lambda)\Phi(\mathbf{x}, \lambda)}{\pi A} \tag{4.1}$$

Here $\Phi(\mathbf{x}, \lambda)$ is the power hitting the surface and A is the area of the surface. This is a useful property because it is sometimes convenient to work in units of power, and to switch later to radiance.

4.1.1 Diffuse Transmission

Some materials, such as paper, exhibit approximately diffuse reflection[1], but also allow some transmission of light. This transmission does not allow the light to pass unscattered, so detail is lost coming through the material. This type of transmission is often called *translucence*. Translucent materials can be modeled as surfaces which diffusely reflect a portion of incoming light, absorb another portion, and diffusely transmit the rest. For light coming from 'above' such a surface, the **BRDF** can be expressed:

$$\rho_t(\psi_{out}, \psi_{in}, \lambda) = \begin{cases} \frac{R(\lambda)}{\pi} & \text{if } 0° < \theta < 90° \\ \frac{T(\lambda)}{\pi} & \text{if } 90° < \theta < 180° \end{cases}$$

The corresponding **SPF** , as shown in Figure 4.2, is:

$$s_t(\psi_{out}, \psi_{in}, \lambda) = \begin{cases} \frac{R(\lambda)cos\theta}{(R(\lambda)+T(\lambda))\pi} & \text{if } 0° < \theta < 90° \\ \frac{-T(\lambda)cos\theta}{(R(\lambda)+T(\lambda))\pi} & \text{if } 90° < \theta < 180° \end{cases}$$

[1] Paper, and many other materials, have strong directionally dependent reflection at extreme angles.

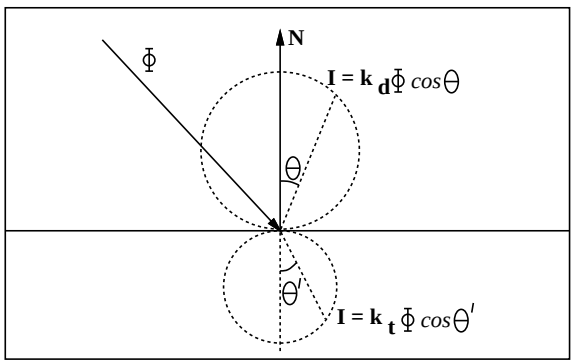

Figure 4.2: Ideal diffuse transmission.

If the incident light comes from 'below' the surface, then we will have the same situation, but in general we might have R' and T' for these angles that are different from R and T. Because T and T' are determined by the 'available paths' that light can take through the material, the reciprocity principle implies that $T = T'$. This observation reduces the two directional transmission coefficients to one common coefficient. If the material is homogeneous in structure (uniform distribution of fibers or grains), then we'd also expect $R = R'$. This will not be the case if one side of the material is somehow different than the other, as is the case for paper that has printing on it.

4.2 Specular Reflection and Refraction

A very commonly used reflective type is the specular surface. Specular surfaces include polished metals, glasses, and any smooth reflective surface. The specular surface is most easily described by its **SPF** which is a delta function times an attenuation term. The attenuation will typically vary with incident angle as discussed in Section 2.2. This is evident in the extreme reflectivity of glass viewed at an acute angle, and its transparency when viewed straight on. As implied by

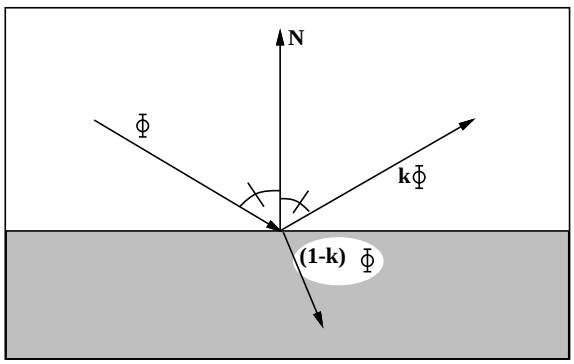

Figure 4.3: Reflection from an ideal specular surface.

Figure 4.3, the specular **SPF** is:

$$s_s(\psi_{out}, \psi_{in}, \lambda) = k_s \delta(\psi_{\mathbf{s}} - \psi_{\mathbf{out}}) + k_t \delta(\psi_{\mathbf{t}} - \psi_{\mathbf{out}})$$

Here k_s and k_t are determined by the Fresnel Equations, δ is the delta function (see Glossary), $\psi_{\mathbf{s}}$ is the direction in which light incident from ψ_{in} reflects in, and $\psi_{\mathbf{t}}$ is the direction light incident from ψ_{in} refracts. If the surface is a metal, the transmitted light will be quickly absorbed.

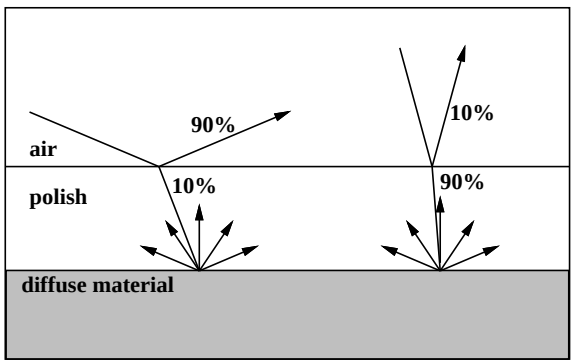

Figure 4.4: Simplified model of polished reflection.

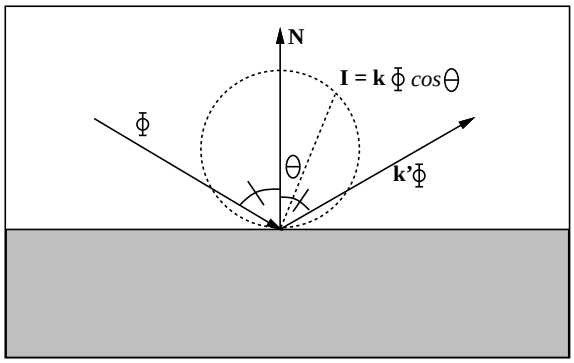

Figure 4.5: Diagram of energy reflection from combined surface.

4.3 Combined Diffuse and Specular Reflection

Many graphics algorithms use a **BRDF** model that is a linear combination of diffuse and specular[121, 44]. If we call the **BRDF** for diffuse ρ_d and the specular **BRDF** ρ_s, the combined **BRDF** ρ_c is:

$$\rho_c = k_d\rho_d + k_s\rho_s$$

This reflection model is usually used to approximate 'polished' surfaces, such as varnished wood. This type of reflector can be thought of as a diffuse reflector covered with a thin dielectric coating. This coating will obey the Fresnel Equations, so it will have variable reflectivity at grazing angles, as shown in Figure 4.4 and Figure 4.5. To account for this variable specularity, k_s can first be calculated using the Fresnel Equations and the refractive index of the polish,

and the **BRDF** will be:

$$\rho_c = [1 - k_s(\theta)]\, k_d \rho_d + k_s(\theta)\rho_s$$

This expression allows k_d and k_s to vary with angle in a natural manner, and allows glare effects found in real scenes. This improvement means that k_s will not have to be 'tuned' if the user wants to model the extreme reflection of polished surfaces when θ is near 90°.

4.4 Glossy Reflection

There are some surfaces that are *not* adequately approximated by diffuse, specular, or combined models. The primary example is brushed metal, which shows some 'fuzzy' reflection. The **BRDF** of these so called *glossy*[26] surfaces can be modeled in several ways.

If the microscopic structure of the surface is known, an analytical model can be used to find the **BRDF** , as was done to derive the Torrance-Sparrow model of reflection[27, 111]. The **BRDF** can also be found by simulated experiment: a model of the surface can be constructed, and the **BRDF** can be approximated by sending many rays from various directions and observing reflections[16].

If the microscopic structure of the surface is not known, then observed values can be used, if available. Otherwise, an empirical model, such as the 'Phong **BRDF** ' used by Immel et al. could be used[56]. It has never been demonstrated that such empirical functions are not adequate for computer graphics applications. In fact, it *has* been demonstrated that empirical functions are sufficient in some contexts. Amanitides' very simplified model of reflection produced excellent pictures of rough metal[3].

It may be much more important to capture surface directional grain than to model a 'accurate' rough reflection function. These *anisotropic* reflection models have been explored by Kajiya[60] and Poulin and Fournier[87].

4.5 Standard Computer Graphics Reflection Models

In many computer graphics programs (such as the Hall model[44]), color is calculated by an equation similar to:

$$C = k_d C_l(\mathbf{N} \cdot \mathbf{L}) + k_s C(\psi_s) + k_t C(\psi_t) + k_h C_l(\psi_s \cdot \mathbf{L})^n$$

Where:

C	Color of surface.
C_l	Color of light source.
$\mathbf{N}$	Unit surface normal vector.
$\mathbf{L}$	Unit vector toward light source.
ψ_s	Unit vector in reflection direction.
ψ_t	Unit vector in transmission direction.
k_d	Diffuse reflectance.
k_s	Specular reflectance.
k_t	Specular transmittance.
k_h	Phong reflectance.
n	Phong exponent.

Here it is assumed that the light source is a 'point light source infinitely far away'. This allows us to use a constant C_l at all points (L_{in} is a constant). The terms each account for a different effect:

$k_d C_l(\mathbf{N} \cdot \mathbf{L})$	Diffusely reflected light.
$k_s C(\psi_\mathbf{s})$	Specularly reflected light. $C(\psi_s)$ is the color seen in direction ψ_s, which is attenuated by the specular reflectance k_s.
$k_t C(\psi_\mathbf{t})$	Specularly transmitted light. $C(\psi_\mathbf{t})$ is the color seen in direction $\psi_\mathbf{t}$,

	which is attenuated by the specular transmittance k_t.
$k_h C_l(\psi_{\mathbf{s}} \cdot \mathbf{L})^n$	Phong highlight term. Gives fuzzy reflection of point light. The Specular term would take care of this for non-point light sources.

For smooth metals, k_s should be the only non-zero constant, and should be set to the normal reflectance of the metal. If ray tracing is not available (so k_s and k_t cannot be used), then k_d and k_h can be set to the normal reflectance of the metal. For clear dielectrics, k_s and k_t should be non-zero constants. If the object is to be viewed from a grazing angle, k_s should be made larger if Fresnel Equations cannot be used. The sum of k_s and k_t should be one. The k_h term can be used if only point light sources are available. For matte surfaces, k_d should be non-zero. For polished surfaces, k_s should also be non-zero.

The chief problems with this model are that there is no global illumination and that k_s does not obey the Fresnel Equations. This means indirect lighting will not be included, and glare will look wrong. Some codes do use the Fresnel Equations[43], but only for pure dielectrics; polished surfaces are restricted to constant k_s.

4.6 Summary

The macroscopic reflectance properties of surfaces can be described by their **BRDF** . The diffuse surface is an approximation to a matte material. The diffuse transmitter is an approximation to the translucent surface, and it was argued that transmission characteristics should be the same in both directions. That observation simplifies the use of the diffuse transmitter model by eliminating one parameter. The specular surface is a smooth dielectric or metal. The combined surface is an approximation to polished materials, and thus the specular term should

follow the Fresnel Equations, and the diffuse term should only reflect light not reflected by the specular term. This angle based interdependence between the diffuse and specular term is not used in previous reflection models, but is vital for correct glare effects. Any reflection behavior not covered by these approximations is called glossy, and such surfaces can have arbitrarily complex **BRDF**s . Standard parametrically driven Computer Graphics reflection models are not sufficient for realistic behavior, but guidelines were given that at least approximate desired behavior.