

ARBITRARY RECURSION

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alternative recursive definition of the measure function:

$\varphi(u) \triangleq$ if $u \in \mathbb{N}$ then u else 0 - fixing function for \mathbb{N}

$\varepsilon_t(\bar{x}) \triangleq \varepsilon k. a(\bar{d}^{\varphi(k)}(\bar{x}))$ - witness of t , for slightly modified definition $t(\bar{x}) \triangleq [\exists k. a(\bar{d}^{\varphi(k)}(\bar{x}))]$

$\nu(\bar{x}, k) \triangleq$ let $\tilde{k} = \varphi(k)$ in if $a(\bar{d}^{\tilde{k}}(\bar{x})) \vee \tilde{k} \geq \varphi(\varepsilon_t(\bar{x}))$ then \tilde{k} else $\nu(\bar{x}, \tilde{k}+1)$ } recursively find min k such that $a(\bar{d}^k(\bar{x}))$, if $t(\bar{x})$; stop at $\varphi(\varepsilon_t(\bar{x}))$ anyhow; min k is always found if $t(\bar{x})$, because $\min k \leq \varphi(\varepsilon_t(\bar{x}))$

$\mu_\nu(\bar{x}, k) \triangleq \varphi(\varepsilon_t(\bar{x}) - \varphi(k))$ $\prec_\nu \triangleq \prec \subseteq \mathbb{N} \times \mathbb{N}$

$\vdash \boxed{\tau_\nu} \neg a(\bar{d}^{\varphi(k)}(\bar{x})) \wedge \varphi(k) < \varphi(\varepsilon_t(\bar{x})) \Rightarrow \mu_\nu(\bar{x}, \varphi(k)+1) \prec_\nu \mu_\nu(\bar{x}, k)$

$\mu_\nu(\bar{x}, \varphi(k)+1) \stackrel{\delta_{\mu_\nu}}{=} \varphi(\varepsilon_t(\bar{x}) - \varphi(\varphi(k)+1)) \stackrel{\delta_\varphi}{=} \varphi(\varepsilon_t(\bar{x}) - \varphi(k) - 1) \stackrel{\delta_\varphi}{=} \varepsilon_t(\bar{x}) - \varphi(k) - 1$

$\varphi(k) < \varphi(\varepsilon_t(\bar{x})) \Rightarrow \varphi(\varepsilon_t(\bar{x})) \neq 0 \xrightarrow{\delta_\varphi} \varepsilon_t(\bar{x}) \in \mathbb{N} \xrightarrow{\delta_\varphi} \varphi(k) < \varepsilon_t(\bar{x}) \rightarrow \varepsilon_t(\bar{x}) - \varphi(k) > 0$

$\mu_\nu(\bar{x}, k) \stackrel{\delta_{\mu_\nu}}{=} \varphi(\varepsilon_t(\bar{x}) - \varphi(k)) \stackrel{\delta_\varphi}{=} \varepsilon_t(\bar{x}) - \varphi(k) \xrightarrow{\delta_\nu} \mu_\nu(\bar{x}, \varphi(k)+1) \prec_\nu \mu_\nu(\bar{x}, k) \xleftarrow{\delta_{\tau_\nu}}$

QED

$\vdash \boxed{\nu \mathbb{N}} \nu(\bar{x}, k) \in \mathbb{N}$

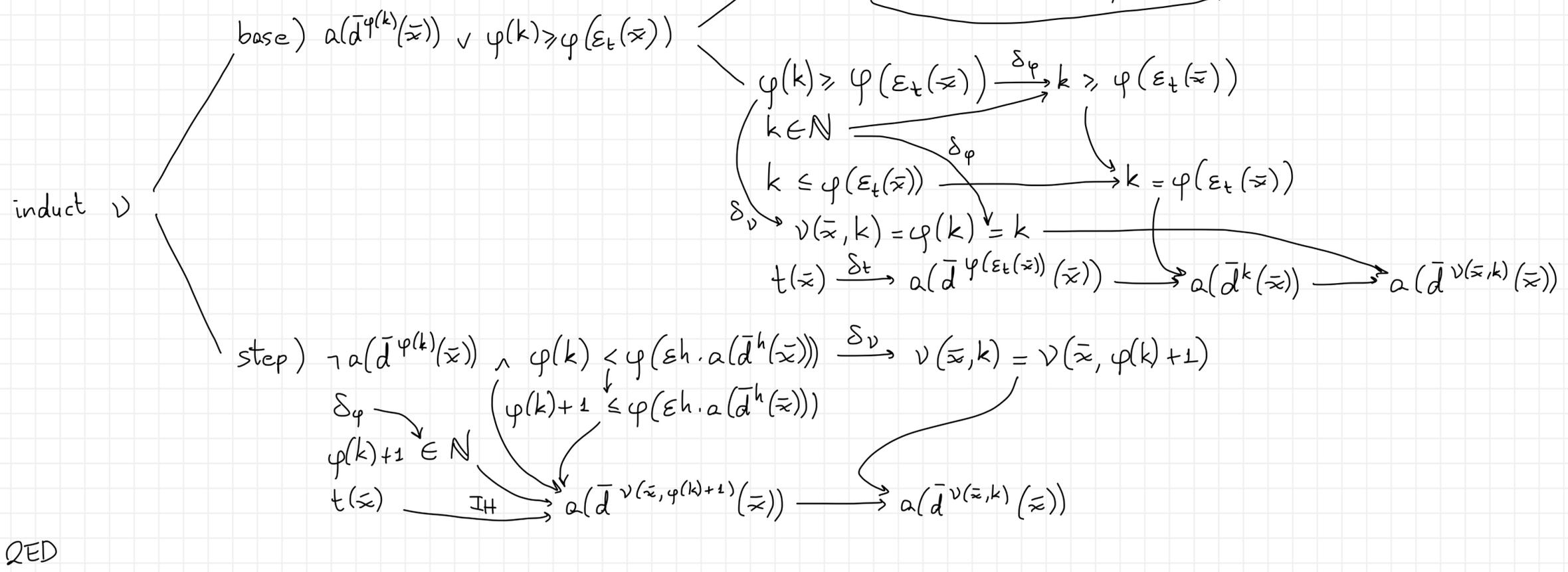
induct ν

- base) $\delta_\nu \rightarrow \nu(\bar{x}, k) = \varphi(k) \rightarrow \nu(\bar{x}, k) \in \mathbb{N}$
 $\delta_\varphi \rightarrow \varphi(k) \in \mathbb{N} \rightarrow \nu(\bar{x}, k) \in \mathbb{N}$
- step) $\delta_\nu \rightarrow \nu(\bar{x}, k) = \nu(\bar{x}, \varphi(k)+1) \rightarrow \nu(\bar{x}, k) \in \mathbb{N}$
 $\text{IH} \rightarrow \nu(\bar{x}, \varphi(k)+1) \in \mathbb{N} \rightarrow \nu(\bar{x}, k) \in \mathbb{N}$

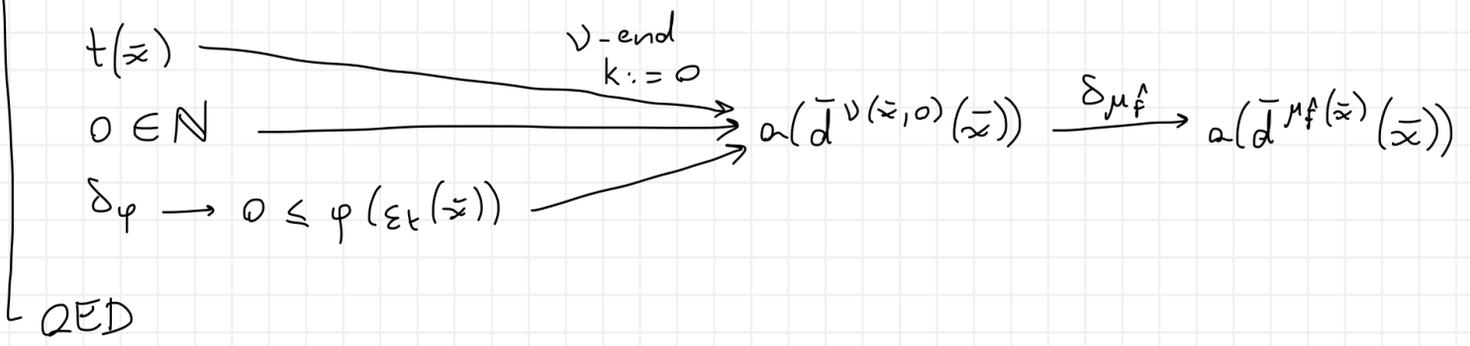
QED

$\mu_{\hat{f}}(\bar{x}) \triangleq \nu(\bar{x}, 0)$ - alternative definition of $\mu_{\hat{f}}$ $\mu_{\hat{f}} : \mathcal{U}^n \rightarrow \mathbb{N} \ (\Leftarrow \nu \mathbb{N})$

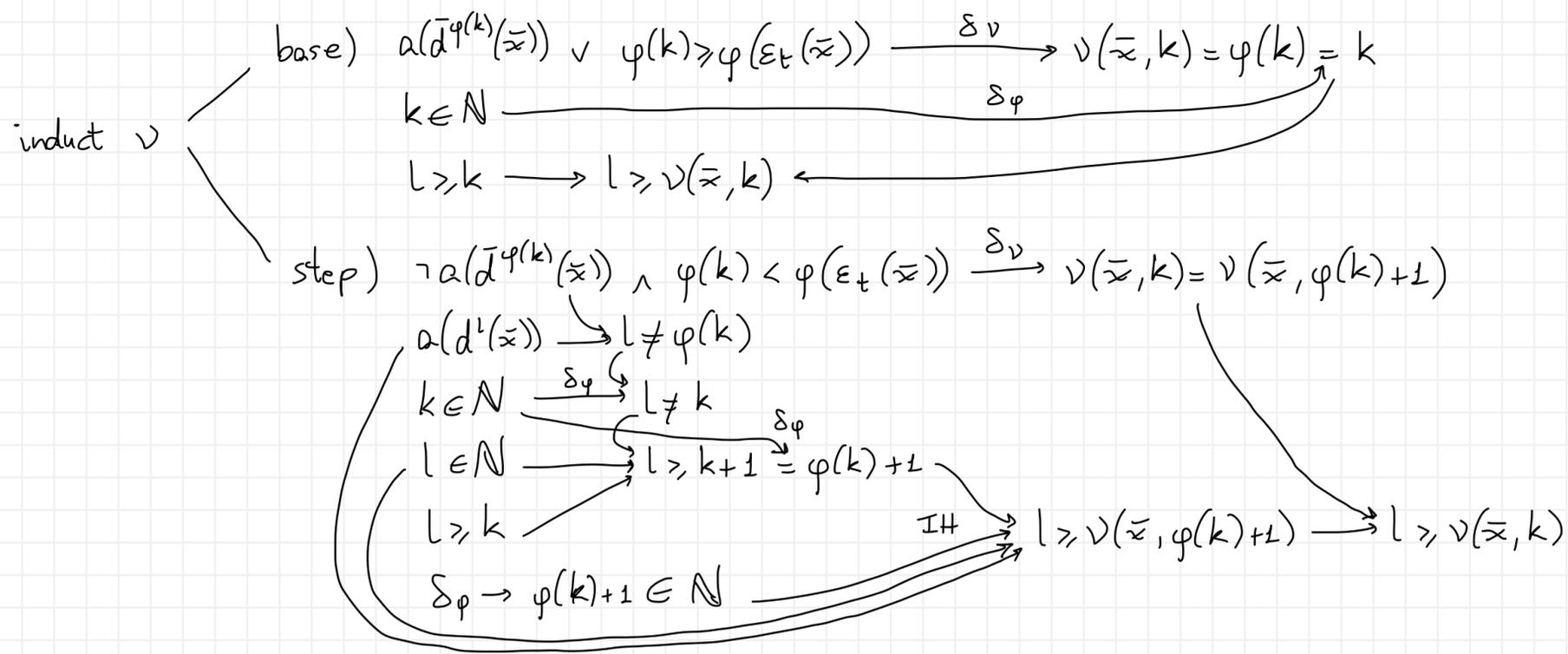
⊢ v-end $t(\bar{x}) \wedge k \in \mathbb{N} \wedge k \leq \varphi(\varepsilon_t(\bar{x})) \Rightarrow a(\bar{d}^{v(\bar{x},k)}(\bar{x}))$



⊢ μ-end $t(\bar{x}) \Rightarrow a(\bar{d}^{\mu_f(\bar{x})}(\bar{x}))$ — alternative proof for the alternative definition of μ_f

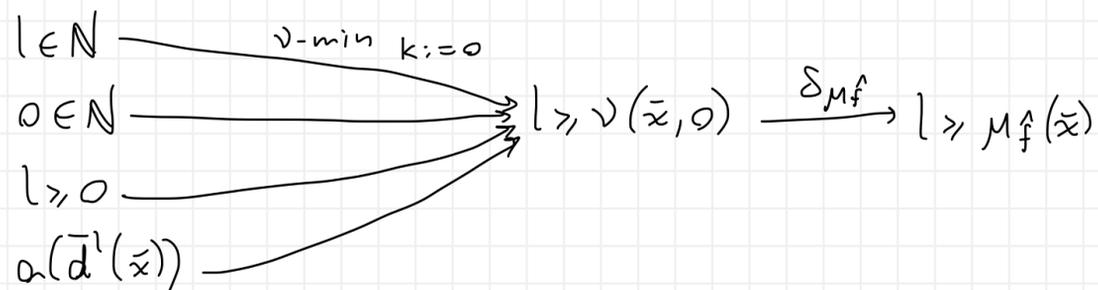


⊢ v-min $l \in \mathbb{N}, k \in \mathbb{N}, l \geq k, a(d^l(\bar{x})) \Rightarrow l \geq v(\bar{x}, k)$



QED

⊢ mu-min $a(d^l(\bar{x})) \Rightarrow l \geq \mu_f(\bar{x})$ — alternative proof for the alternative definition of μ_f



QED

τ_f proved as before using μ -end and μ -min